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# An alternating direction (ADI) method for a self-acting rectangular gas bearing

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SUMMARY

The stationary Reynolds equation is solved over a rectangular region. The problem is linearized by Picard linearization. The ADI method is used to solve the resulting set of linear equations. A set of parameters is introduced to speed up convergence as well for the Picard linearization as for the ADI method. A comparison is made with Booy–Coleman's method. Results are given for bearing numbers 10 to 1000.

## 1. Introduction

In gas-bearing problems the so-called Reynolds equation has to be solved numerically, for details see [1]. In this equation the ratio between viscous and non-viscous forces is given by the bearing number $\Lambda$.

For low and moderate values of $\Lambda$ many numerical methods iterative and direct are available. For large values of $\Lambda$ ($\Lambda > 100$) computing time increases and direct methods are preferred to iterative methods. A commonly used direct method is the Booy–Coleman method [2, 3]. This method is rather complex in programming. It needs a large memory space and with increasing $\Lambda$ the number of orthogonalizations increases and so the computing time.

Although the ADI method is not a new approach in gas-bearing problems [1, 5] the method here described uses a set of parameters to accelerate convergence. The acceleration parameters for the ADI method are so chosen that it reduces to a direct method. In comparison with the Booy–Coleman method the here described method is simple to program, uses less memory and is, especially for high values of $\Lambda$ more efficient.

Computations are made for $\Lambda$ in the range 10 to 1000, although it is not necessary for $\Lambda$ to be in this range. In the computations the number of mesh points varies from about 100 to 1000.

## 2. Analysis of the problem

Figure 1 gives a sketch of a rectangular gas bearing with bearing width $B$ and bearing length $L$. The sliding velocity is assumed to be $-\boldsymbol{i}U_0$, where $\boldsymbol{i}$ indicates a positive unit vector.

We assume in the bearing viscous, isothermal and laminar flow so the density $\rho(X, Y)$ is proportional to the pressure $P(X, Y)$, and the viscosity $\sigma$ is constant. From Fig. 1 it follows that $H=(H_0/X_0)X$, if we assume the stationary case. The pressure $P(X, Y)$ can be calculated by solving the Reynolds equation for $P$.

This equation reads in the instationary case [1]:

$$\operatorname{div}\left\{\frac{\rho H^3}{12\sigma}\operatorname{grad} P-\tfrac{1}{2}\rho H U\right\}-\frac{\partial}{\partial t}\rho H=0\,.$$

In our case it reduces to

$$\frac{1}{12\sigma}\operatorname{div}(PH^3\operatorname{grad} P)+\frac{U_0}{2}\frac{\partial(PH)}{\partial X}=0\,,$$

with $P=P_a$ as the boundary condition, where $P_a$ is the ambient pressure.

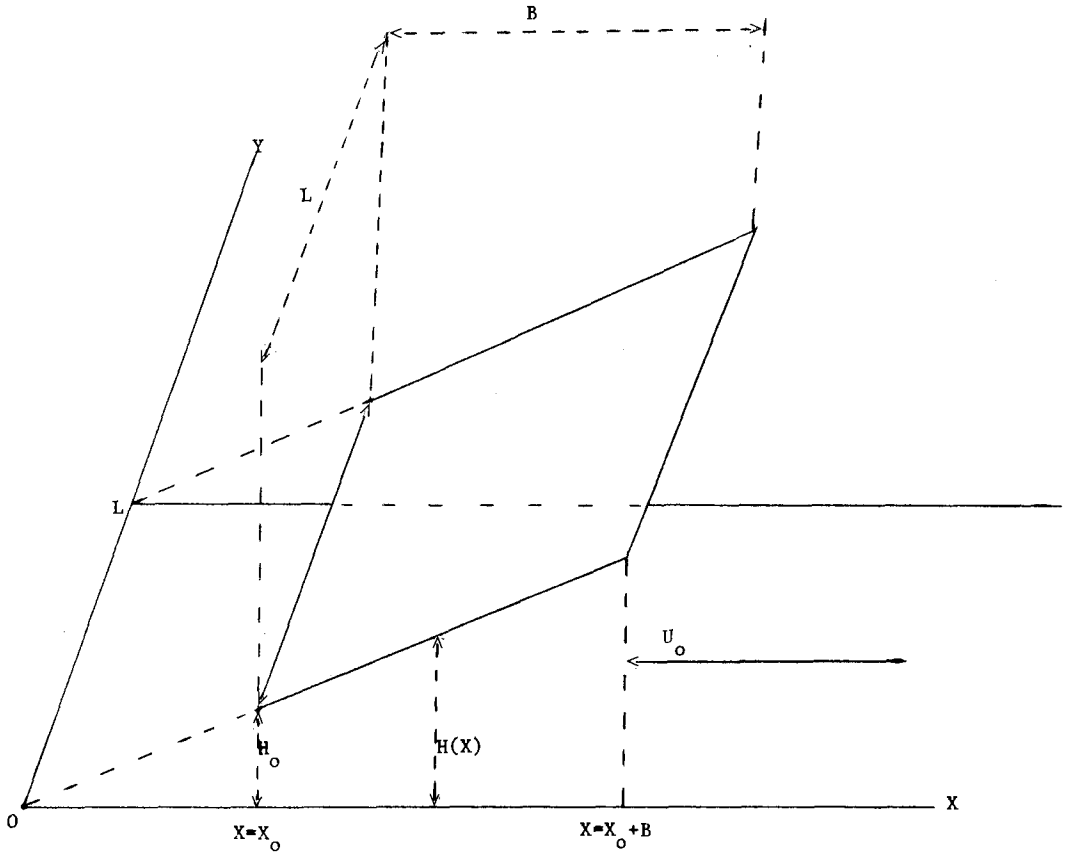


Figure 1. Plane rectangular slider bearing schematic.

Introducing the dimensionless quantities

$$p = \frac{P}{P_a}, \quad x = \frac{X}{X_0}, \quad y = \frac{Y}{X_0}, \quad h = \frac{H}{H_0},$$

the Reynolds equation reduces to:

$$\frac{\partial}{\partial x}\left(px^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(px^3 \frac{\partial p}{\partial y}\right) + 6 \frac{U_0 \sigma X_0}{H_0^2 P_a} \frac{\partial (px)}{\partial x} = 0 .$$

Using the definition $Q = p^2 x^2$ and $\Lambda = 6U_0 \sigma X_0/(H_0^2 P_a)$, the Reynolds equation can be written

$$\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} - \frac{1}{x}\frac{\partial Q}{\partial x} = -\frac{\Lambda}{x Q^{\frac{1}{2}}} \frac{\partial Q}{\partial x} \tag{1}$$

with boundary conditions:

$$\left.\begin{aligned} &Q(1, y) = 1 \\ &Q\left(1 + \frac{B}{X_0}, y\right) = \left(1 + \frac{B}{X_0}\right) \end{aligned}\right\} \quad 0 \leqq y \leqq L/2 ,$$

$$\left.\begin{aligned} &Q(x, 0) = x^2 \\ &Q_y|_{y=L/2} = 0 \end{aligned}\right\} \quad 1 \leqq x \leqq 1 + B/X_0 .$$

where $Q_y$ indicates differentiation with respect to $y$. The condition $Q_y|_{y=L/2} = 0$ is a consequence of the symmetry of the problem.

### 3. Description of the method of solution

In order to solve the problem use will be made of an iterative method with some special features. Therefore instead of the non-linear equation (1) we consider the linearized equation:

$$Q_{xx}^{(k)}+Q_{yy}^{(k)}+(-1/x+\mu)Q_x^{(k)}-\omega Q^{(k)}=Q_x^{(k-1)}\left(\frac{-\Lambda}{x(Q^{(k-1)})^{\frac{1}{2}}}+\mu\right)-\omega Q^{(k-1)} \tag{2}$$

with $k=1, 2, 3, \ldots$, and $Q_x$ and $Q_y$ etc. indicates differentiation with respect to $x$ and $y$, respectively. Estimates for the starting value $Q^{(0)}$ and the parameters $\mu$ and $\omega$ will be given in the sequel.

Discretizing with the normal 5-point molecule gives

$$Q_{xx}=\frac{Q_{i-1,j}-2Q_{i,j}+Q_{i+1,j}}{h_x^2}+O(h_x^2),$$

$$Q_{yy}=\frac{Q_{i,j-1}-2Q_{i,j}+Q_{i,j+1}}{h_y^2}+O(h_y^2),$$

$$Q_x=\frac{Q_{i+1,j}-Q_{i-1,j}}{2h_x}+O(h_x^2),$$

In Fig. 2 the meshsize and the number of meshpoints is given.

Defining $\alpha=2(1+\gamma)+\omega h_x^2$, and $\beta_i=h_x(\mu-1/x_i)/2$, gives for the discretized form of equation (2):

$$(-1+\beta_i)Q_{i-1,j}^{(k)}+\alpha Q_{i,j}^{(k)}+(-1-\beta_i)Q_{i+1,j}^{(k)}-\gamma Q_{i,j-1}^{(k)}-\gamma Q_{i,j+1}^{(k)}=-h_x^2U_{i,j}^{(k-1)}$$

with $\gamma=h_x^2/h_y^2$ and $i=1(1)M-2$, $j=0(1)N-2$,

$$U_{i,j}^{(k-1)}=\left(\frac{-\Lambda}{x_i(Q_{i,j}^{(k-1)})^{\frac{1}{2}}}+\mu\right)\left(\frac{Q_{i+1,j}^{(k-1)}-Q_{i-1,j}^{(k-1)}}{2h_x}\right)-\omega Q_{i,j}^{(k-1)}.$$

The boundary conditions become:

$$Q(0,j)=1,$$

$$Q(i,N-1)=x_i^2,$$

$$Q(M-1,j)=\left(1+\frac{B}{X_0}\right)^2,$$

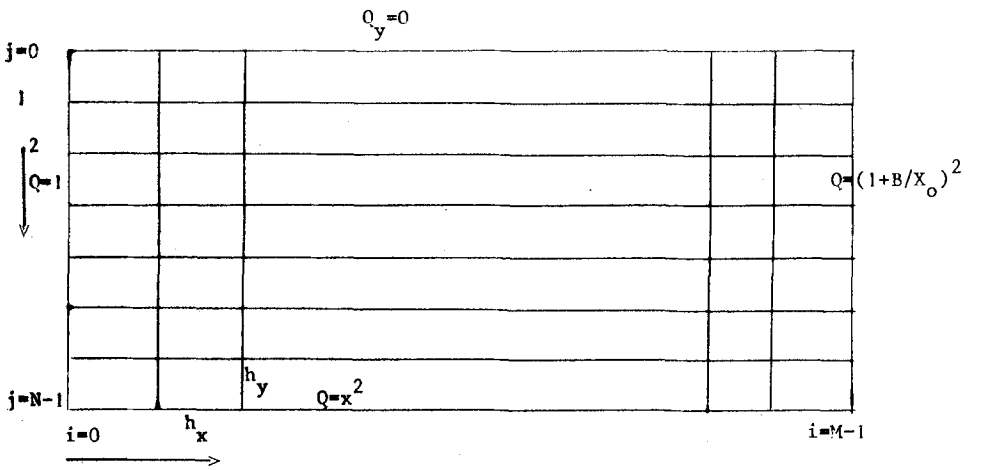


Figure 2. The rectangular integration region, $h_x$ and $h_y$ sides of the rectangular meshes. $N$ = number of horizontal lines. $N$ = number of vertical lines.

and $Q(i, 1) = Q(i, -1)$.

The discretized problem gives rise to a set of linear equations $AQ = F$.

The matrix $A$ is a sparse matrix and the non-zero elements can be composed by the following rules:

$$
\begin{aligned}
&a_{ii} = \alpha, && i = 1, 2, \ldots (N-1)(M-2),\\
&a_{i,i+1} = -\gamma, && i = 2, 3, \ldots, N-1, N+1, \ldots, 2N-2, 2N, \ldots (N-1)(M-2)-1,\\
&a_{i,i+1} = -2\gamma, && i = 1, N, 2N-1, 3N-2, \ldots, (M-3)(N-1)+1,\\
&a_{i,i-1} = -\gamma, && i = 2, 3, \ldots N-1, N+1, \ldots, 2N-2, 2N, \ldots (N-1)(M-2),\\
&a_{i,i-1} = 0, && i = N, 2N-1, 3N-2, \ldots, (M-3)(N-1)+1,\\
&a_{i,N+i-1} = -1-\beta_p, && i = 1, 2, \ldots (N-1)(M-3) \text{ and } p = \text{entire}\left(\frac{i-1}{N-1} + 1\right),\\
&a_{i+1,i-N+2} = -1+\beta_p, && i = N-1, N, N+1, \ldots, (N-1)(M-2)-1\\
& && \text{and } p = \text{entire}\left(\frac{i}{N-1} + 1\right).
\end{aligned}
$$

All the other elements are zero and $\boldsymbol{F}$ is defined by:

$$
\boldsymbol{F} = \begin{bmatrix}
-h_x^2 U_{1,0} + 1 - \beta_1\\
-h_x^2 U_{1,1} + 1 - \beta_1\\
\vdots\\
-h_x^2 U_{1,N-1} + 1 - \beta_1 - x_1^2\gamma\\
-h_x^2 U_{2,0}\\
\vdots\\
-h_x^2 U_{2,N-1} + x_2^2\gamma\\
\vdots\\
-h_x^2 U_{M-2,N-1} + x_{M-2}^2\gamma + \delta
\end{bmatrix}
$$

where

$$\delta = (1+\beta_{M-2})\left(1 + \frac{B}{X_0}\right)^2.$$

According to the ADI method [4] the matrix $A$ is splitted in the matrices $\overline{H}$ and $\overline{V}$, so $A = \overline{H} + \overline{V}$, where

$$
\overline{H} = \begin{bmatrix}
\alpha/2 & -1-\beta_1 & & & & & & & & & & & \\
 & \ddots & \ddots & & & & & & & & & & \\
 & & \alpha/2 & -1-\beta_1 & & & & & & & & & \\
 & & -1+\beta_2 & \alpha/2 & -1-\beta_2 & & & & & & & & \\
 & & & \ddots & \ddots & \ddots & & & & & & & \\
 & & & & -1+\beta_2 & \alpha/2 & -1-\beta_2 & & & & & & \\
 & & & & & \ddots & \ddots & \ddots & & & & & \\
 & & & & & & -1+\beta_2 & \alpha/2 & -1-\beta_2 & & & & \\
 & & & & & & & \ddots & \ddots & \ddots & & & \\
 & & & & & & & & -1+\beta_{M-3} & \alpha/2 & -1-\beta_{M-3} & & \\
 & & & & & & & & & \ddots & \ddots & \ddots & \\
 & & & & & & & & & & -1+\beta_{M-2} & \alpha/2 & -1-\beta_{M-2} \\
 & & & & & & & & & & & \ddots & \alpha/2
\end{bmatrix}
$$

The blocksize of the blocks in $\overline{H}$ is $N-1$.

If we define the matrix $W$ to be:

$$W = \begin{bmatrix} \alpha/2 & -2\gamma & & & & \\ -\gamma & \alpha/2 & -\gamma & & & \\ & -\gamma & \alpha/2 & -\gamma & & \\ & & \ddots & \ddots & \ddots & \\ & & & -\gamma & \alpha/2 & -\gamma \\ & & & & -\gamma & \alpha/2 \end{bmatrix} \text{ with rank } N-1 .$$

then $\overline{V}$ can be written as a diagonal block matrix:

$$\overline{V} = \begin{bmatrix} W & & & \\ & W & & 0 \\ & & W & \\ & & & \ddots \\ 0 & & & W \end{bmatrix}$$

So the $M-2$ uncoupled matrices $W$ compose the matrix $\overline{V}$.

In the ADI method a set of acceleration parameters has to be chosen. Calling these parameters $r_m$ then the system $AQ=F$ can be solved with the following iterative scheme [4]:

$$\left.\begin{aligned} (\overline{H}+r_m I)\boldsymbol{Q}^{(s+\frac{1}{2})} &= (r_m I-\overline{V})\boldsymbol{Q}^{(s)}+\boldsymbol{F} \\ (\overline{V}+r_m I)\boldsymbol{Q}^{(s+1)} &= (r_m I-\overline{H})\boldsymbol{Q}^{(s+\frac{1}{2})}+\boldsymbol{F} \end{aligned}\right\} \tag{3}$$

The so-called Peaceman–Rachford matrix is defined by:

$$T_r = (\overline{V}+rI)^{-1}(rI-\overline{H})(\overline{H}+rI)^{-1}(rI-\overline{V})$$

and the error in the $s$th iterationstep by $\boldsymbol{\varepsilon}^{(s)}=\boldsymbol{Q}^{(s)}-\boldsymbol{Q}$. So

$$\boldsymbol{\varepsilon}^{(s)} = \prod_{p=1}^{s} T_{r_p}\boldsymbol{\varepsilon}^{(0)} .$$

In linearizing equation (1) we have chosen for the so-called Picard linearization.

By considering equation (2), it is seen that the matrices $-1/h_x^2\,\overline{H}$ and $-1/h_y^2\,\overline{V}$ are the discretized forms of the operators

$$\frac{\partial^2}{\partial x^2} + (-1/x+\mu)\frac{\partial}{\partial x} - \omega/2 \text{ and } \frac{\partial^2}{\partial y^2} - \omega/2 .$$

By direct calculation it follows that these operators do not commute if $\mu=\mu(x, y)$ or $\omega=\omega(x, y)$. A sufficient condition for the matrices $\overline{H}$ and $\overline{V}$ to commute is that $\omega$ and $\mu$ are constants.

It is easy to see that if the matrices $\overline{H}$ and $\overline{V}$ commute, the following matrices commute:

$$(\overline{V}+rI)^{-1},\ (rI-\overline{H}),\ (\overline{H}+rI)^{-1} \text{ and } (rI-\overline{V}) .$$

Now we are able to look for a set of acceleration parameters $\{r_m\}$. Suppose $\overline{V}$ with rank $\mathcal{M}$ possesses a set of $\mathcal{M}$ linear independent eigenvectors $\{\boldsymbol{t}_k\}$ and corresponding eigenvalues $\{v_k\}$. Then

$$\boldsymbol{\varepsilon}^{(0)} = \sum_{k=1}^{\mathcal{M}} \alpha_k \boldsymbol{t}_k , \quad \boldsymbol{\varepsilon}^{(s)} = \prod_{j=1}^{s} T_{r_j} \sum_{k=1}^{\mathcal{M}} \alpha_k \boldsymbol{t}_k$$

or

$$\boldsymbol{\varepsilon}^{(s)} = \sum_{k=1}^{\mathcal{M}} \alpha_k \prod_{j=1}^{s} T_{r_j}\boldsymbol{t}_k = \sum_{k=1}^{\mathcal{M}} \alpha_k \tilde{H}\boldsymbol{t}_k \prod_{j=1}^{s} \frac{r_j - v_k}{r_j + v_k}$$

with

$$\tilde{H} = (r_s I - \bar{H})(\bar{H} + r_s I)^{-1}(r_{s-1} I - \bar{H})(\bar{H} + r_{s-1} I) \ldots (r_1 I - \bar{H})(\bar{H} + r_1 I)^{-1} .$$

The following inequality holds:

$$\|\varepsilon^{(s)}\| \leqq \sum_{k=1}^{\mathcal{M}} |\alpha_k| \, \|\tilde{H} t_k\| \prod_{j=1}^{S} \frac{r_j - v_k}{r_j + v_k} .$$

By choosing $r_j = v_j$, $\|\varepsilon^{(s)}\| = 0$ after $\mathcal{M}$ iterations.

$\bar{V}$ is completely reducible and is composed of $M-1$ matrices $W$ with rank $N-1$. The iteration scheme (3) reduces after $N-1$ iterations to a direct method. To get the above mentioned method we have to calculate the eigenvalues of $W$. Instead of $W$, we consider the matrix $\tilde{W}$,

$$\tilde{W} = \begin{bmatrix} \alpha/2 & -\gamma & & & \\ -\gamma & \alpha/2 & -\gamma & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & -\gamma \\ & & & -\gamma & \alpha/2 \end{bmatrix}, \quad \text{with rank } 2N-3 .$$

It is easy to see that the $k$th component of the $q$th eigenvector of $W$ can be written as $d \sin[q\Pi k/(2N-2)]$, with $d$ a constant ($k = 1, 2, \ldots, 2N-3$).

The first row of $W$ is situated at the $(N-1)$th row of $\tilde{W}$. So we are looking for eigenvectors of $\tilde{W}$ with components $d \sin[q\Pi k/(2N-2)]$ with $k = N-1, N, \ldots, 2N-3$. Substitution of the so formed eigenvectors $\boldsymbol{t}$ in the matrix vector product $W\boldsymbol{t}$, gives as eigenvalues for $W$,

$$-2\gamma \cos \frac{q\Pi}{2N-2} + \alpha/2 ,$$

with $q = 2p-1$ and $p = 1(1)N-1$.

## 4. Estimates for $\omega$ and $\mu$

In equation (1) the parameters $\mu$ and $\omega$ are introduced to speed up the convergence of problem (2).

We define $z^{(k)} = Q^{(k)} - Q^{(k-1)}$, $k \geqq 2$. Using the definition

$$C(k) = \frac{-\Lambda}{x(Q^{(k)})^{\frac{1}{2}}} + \mu ,$$

it follows from (2) that

$$z_{xx}^{(k)} + z_{yy}^{(k)} + (\mu - 1/x) z_x^{(k)} - \omega z^{(k)} = C(k-1) Q_x^{(k-1)} - C(k-2) Q_x^{(k-2)} - \omega z^{(k-1)} ,$$

with $z = 0$ on the boundary of the rectangle $G\{x, y \mid 1 \leqq x \leqq 1 + B/X_0, 0 \leqq y \leqq L\}$. We expand $(Q_x/(Q)^{\frac{1}{2}})^{(k-1)}$ in the function space $\{Q^{(k)}\}$ about the "point" $Q^{(k-2)}$, neglecting second order terms. This gives:

$$z_{xx}^{(k)} + z_{yy}^{(k)} + (\mu - 1/x) z_x^{(k)} - \omega z^{(k)} = C(k-2) z_x^{(k-1)} + \left( \frac{\Lambda Q_x^{(k-2)}}{2xQ^{(k-2)}(Q^{(k-2)})^{\frac{1}{2}}} - \omega \right) z^{(k-1)} .$$

This equation is an elliptic equation which follows the maximum principle if $\omega > 0$ [7, p. 88]. So

$$\max_G |z^{(k)}| \leqq K \max_G \left( C(k-2) z_x^{(k-1)} + \left( \frac{\Lambda Q_x^{(k-2)}}{2xQ^{(k-2)}(Q^{(k-2)})^{\frac{1}{2}}} - \omega \right) z^{(k-1)} \right) .$$

As $Q$ is bounded on $G$ if $\Lambda \to \infty$ we choose for

$$\mu = \tfrac{1}{2} \left( \min_G \frac{\Lambda}{x(Q^{(k-2)})^{\frac{1}{2}}} + \max_G \frac{\Lambda}{x(Q^{(k-2)})^{\frac{1}{2}}} \right) = \Lambda/2(1 + 1/(1 + B/X_0)^2) ,$$

to minimize $C(k-2)$.

If $\Lambda \to \infty$, $Q_x$ is not bounded everywhere, so it is not possible to get a feasible estimate for $\omega$. However if $\Lambda \to \infty$ we can make the assumption that $Q$ is independent of $y$ (see the next section) and as an estimation for $Q_x$ we use for $\Lambda \gg 1$, the mean value of the asymptotic value of $Q_x$ for $\Lambda \to \infty$ over the interval $1 \leqq x \leqq 1 + B/X_0$. So

$$\omega = \Lambda/4\,(1+(1+B/X_0)^2)/(B/X_0)\,(1+1/(1+B/X_0)^4)\,.$$

## 5. Estimation of the starting value $Q^{(0)}$

A commonly used estimate for $Q^{(0)}$ is $Q^{(0)} = x^2$. This is a feasible estimation for small values of the bearing number $\Lambda$. For large values of $\Lambda$ we assume that, except for a boundary layer $0 \leqq y \leqq \delta$, $Q$ is independent of $y$.

Using the definition $B_0 = (1 + B/X_0)$, $Q$ tends to $B_0^2$ if $\Lambda$ tends to infinity, and equation (1) is approximated by

$$Q_{xx} = -\Lambda/(B_0^2)\,Q_x/x\,,$$

with boundary conditions $Q = 1$ if $x = 1$ and $Q = B_0^2$ if $x = B_0$.

Solving this equation gives

$$Q = Q_b = \frac{(B_0^2 - 1)(x^{-\Lambda/B_0 + 1} - 1)}{B_0^{-\Lambda/B_0 + 1} - 1} + 1\,. \tag{4}$$

In the boundary layer (1) is approximated by $Q_{yy} = -2\Lambda/B_0$ and the boundary conditions are

$$\begin{aligned} Q_y &= 0 \\ Q &= Q_b \end{aligned} \qquad y = \delta\,, \text{ and } Q = x^2 \text{ if } y = 0\,.$$

So we get

$$Q = -\Lambda/B_0\,y^2 + 2\Lambda\delta/B_0\,y + x^2 \tag{5}$$

and $\delta = (B_0(Q_b - x^2)/\Lambda)^{\frac{1}{2}}$.

We use (4) as a starting value if $\delta \leqq y \leqq L/2$ and (5) if $0 < y < \delta$.

## 6. Numerical results

TABLE 1

*Pressure distribution for* $\Lambda = 10$, $(Q^{\frac{1}{2}}/x) - 1 = p$.

| $X$ | BC ($M=21$, $N=7$) | ADI ($M=21$, $N=7$) | ADI ($M=41$, $N=13$) |
|---|---|---|---|
| 1.05 | 0.1520 | 0.1525 | 0.1527 |
| 1.10 | 0.2176 | 0.2183 | 0.2184 |
| 1.15 | 0.2441 | 0.2449 | 0.2450 |
| 1.20 | 0.2504 | 0.2513 | 0.2512 |
| 1.25 | 0.2454 | 0.2464 | 0.2463 |
| 1.35 | 0.2194 | 0.2205 | 0.2204 |
| 1.40 | 0.2029 | 0.2040 | 0.2039 |
| 1.45 | 0.1854 | 0.1866 | 0.1864 |
| 1.50 | 0.1676 | 0.1688 | 0.1687 |
| 1.55 | 0.1499 | 0.1511 | 0.1510 |
| 1.60 | 0.1323 | 0.1335 | 0.1334 |
| 1.65 | 0.1150 | 0.1162 | 0.1161 |
| 1.70 | 0.0979 | 0.0992 | 0.0991 |
| 1.75 | 0.0811 | 0.0824 | 0.0823 |
| 1.80 | 0.0645 | 0.0658 | 0.0657 |
| 1.85 | 0.0480 | 0.0493 | 0.0493 |
| 1.90 | 0.0316 | 0.0329 | 0.0329 |
| 1.95 | 0.0152 | 0.0165 | 0.0165 |

The calculations were made on the IBM 360/50 computer of the T.H. Twente. The results of this method are compared with the results of the Booy–Coleman method. In the following tables the pressure computed by Booy–Coleman (BC) and ADI are shown for the line $y = L/2$, and $\Lambda = 10$ and 100.

The programs are written in Fortran, using single precision for the Booy–Coleman method and double precision for the ADI method.

The dimensions are taken $L = B = X_0 = 1$.

TABLE 2

*Pressure distribution for* $\Lambda = 100$.

| $X$ | BC ($M=21$, $N=7$) | ADI ($M=21$, $N=7$) | ADI ($M=41$, $N=7$) | ADI ($M=81$, $N=7$) |
|---|---|---|---|---|
| 1.05 | 0.9331 | 0.9352 | 0.8484 | 0.8363 |
| 1.10 | 0.7981 | 0.8003 | 0.8004 | 0.7986 |
| 1.15 | 0.7246 | 0.7266 | 0.7263 | 0.7260 |
| 1.20 | 0.6543 | 0.6561 | 0.6561 | 0.6561 |
| 1.25 | 0.5896 | 0.5913 | 0.5913 | 0.5913 |
| 1.30 | 0.5299 | 0.5314 | 0.5314 | |
| 1.35 | 0.4745 | 0.4758 | 0.4758 | |
| 1.40 | 0.4229 | 0.4241 | 0.4241 | |
| 1.45 | 0.3748 | 0.3759 | 0.3759 | |
| 1.50 | 0.3298 | 0.3308 | 0.3308 | |
| 1.55 | 0.2876 | 0.2885 | 0.2885 | |
| 1.60 | 0.2480 | 0.2487 | 0.2487 | |
| 1.65 | 0.2106 | 0.2112 | 0.2112 | |
| 1.70 | 0.1754 | 0.1759 | 0.1759 | |
| 1.75 | 0.1421 | 0.1425 | 0.1425 | |
| 1.80 | 0.1106 | 0.1109 | 0.1109 | |
| 1.85 | 0.0807 | 0.0810 | 0.0810 | |
| 1.90 | 0.0524 | 0.0526 | 0.0526 | |
| 1.95 | 0.0255 | 0.0256 | 0.0256 | |

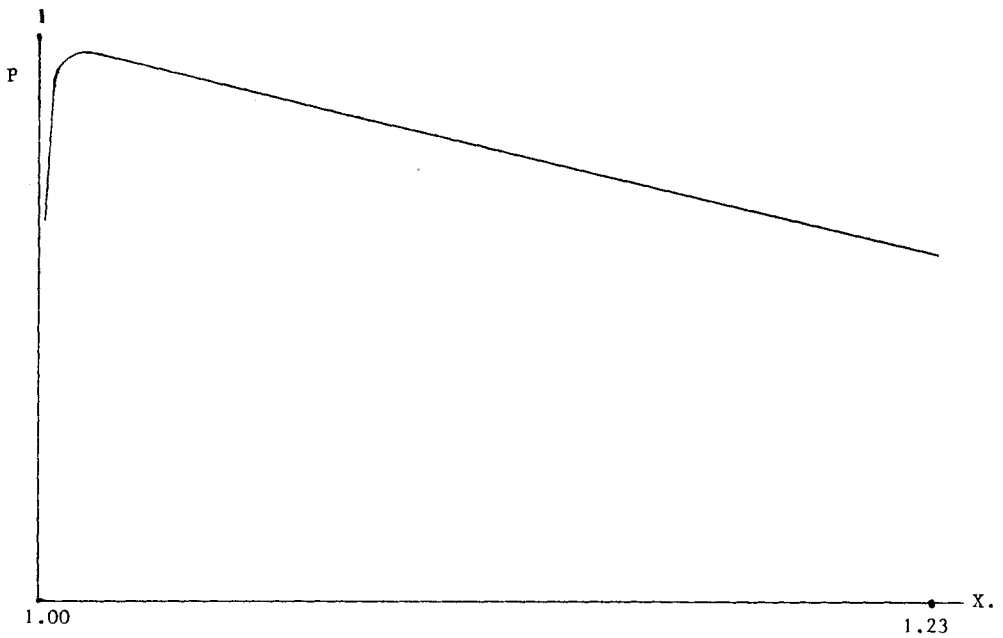


Figure 3. Pressure distribution for $\Lambda = 1000$.

In the Booy–Coleman method the nonlinear problem was linearized with the so-called quasi-linearization. So convergence was quadratic. Number of iterations for BC was 2 for ADI 10.

Computing time was slightly in favour of ADI, 24 sec. to 28 sec. CPU for BC. For higher values of $\Lambda$, the computing time was highly in favour of ADI, although the ADI computations are in double precision.

In Table 2 we see the pressure distribution for $\Lambda = 100$, computing time for BC 101 sec. CPU, for ADI resp. 40 sec., 67 sec. and 191 sec. CPU.

From Table 2 we see that it is difficult to get accurate results near $x = 1$ for higher values of $\Lambda$. Because $x > 1$,

$$x^{-\Lambda/[1+(B/X_0)]+1} \to 0 \quad \text{if} \quad \Lambda \to \infty .$$

In the program, a test is made if

$$(1+\delta_x)^{-\Lambda/[1+(B/X_0)]+1} = 10^{-10}$$

and $\delta_x > 10^{-4}$. If true then $5\delta_x$ is used as integration interval in the $x$-direction. For $x > 5\delta_x$ we use the solution $Q = (1 + B/X_0)^2$. Now we are able to get accurate results near $x = 1$ for high values of $\Lambda$, without increasing the number of mesh points.

The pressure distribution for $\Lambda = 1000$, $\delta_x = 0.047$ is shown in Fig. 3. Only the interval $1 \leqq x \leqq 1.23$ is plotted, for $x > 1.23$ we use the solution $Q = 4$.

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